



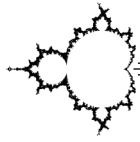
A list of practice problems is provided below to aid in preparation for round one of the 2009 Mandelbrot Team Play. Note that these problems are not meant so serve as a precise indicator of the problems that will appear on the contest. However, students who understand how to solve them should be able to make significantly more progress than they might have otherwise. So work hard on the problems, and good luck on the Team Play!

## Practice Problems

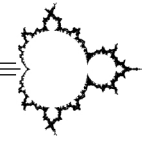
Imagine that parallel rays of light travel across the plane from the right and strike the  $y$ -axis. Then an object to the right of the  $y$ -axis will cast a shadow. Furthermore, for a given object, the length of the shadow will depend on the slope of the light rays. In the following problems you will investigate how the shape of the object affects the length of the shadow and vice-versa.

1. Consider the rectangle with vertices at  $(1, 1)$ ,  $(2, 1)$ ,  $(1, 3)$  and  $(2, 3)$ . What length shadow is cast on the  $y$ -axis if the light rays have slope 0? What about slopes 1,  $-2$  or  $\frac{1}{3}$ ?
2. Let  $R$  be a rectangular region having sides parallel to the  $y$ -axis. Suppose that light rays with slope 2 cast a shadow of length 13 while light rays with slope 5 cast a shadow of length 31. Determine the dimensions of the rectangle.
3. Suppose an object casts a shadow of length 3 regardless of the slope of the light rays. Describe, as precisely as possible, the shape of the object.
4. Let  $A$ ,  $B$ ,  $C$  and  $D$  be four points, and let  $A'$ ,  $B'$ ,  $C'$  and  $D'$  be their shadows on the  $y$ -axis. Depending on the slope of the light rays, the images  $A'$ ,  $B'$ ,  $C'$  and  $D'$  can appear in various orders on the  $y$ -axis (from bottom to top). What is the maximum possible number of different orders that can be obtained?
5. Is it possible for a region to cast a shadow with slope 0 light rays that is longer than either of the shadows cast by slope 1 or  $-1$  light rays?

Don't peek yet! Solutions on the next page.  $\implies$



Team Play Topics  
SOLUTIONS TO PRACTICE PROBLEMS



ROUND ONE

1. The shadows cast will have length 2, 3, 4 and  $2\frac{1}{3}$ . In general, light rays with slope  $m$  will create a shadow of length  $2 + |m|$ .
2. First show that if the rectangle has length  $a$  and height  $b$ , then light rays with slope  $m$  create a shadow of length  $b + ma$ . Hence we know that  $b + 2a = 13$  while  $b + 5a = 31$ , which gives dimensions  $a = 6$  and  $b = 1$ .
3. The only possibility for such a region is a segment of length 3 parallel to the  $y$ -axis. One can prove this by arguing that if there is any path through the region which is not parallel to the  $y$ -axis, then the shadow it casts will eventually become arbitrarily long as the slope goes to infinity.
4. One can obtain seven different orders at the most. This is because the only time the order of the image points changes as the slope varies from negative infinity to infinity is when the parallel light rays pass through two of the points simultaneously. There are at most six such slopes, leading to seven possible orders.
5. I don't believe so, but I'm not completely sure about this. If you find a counterexample, be sure to contact us at [info@mandelbrot.org](mailto:info@mandelbrot.org).