March 2009

⋆ National Level ⋆

The Mandelbrot Competition

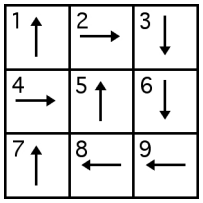
Round Five Test

Time Limit: 40 minutes

Name: _____

1. Determine the value of $\dfrac{1+2+3+\cdots+19}{2+3+4+\cdots+20}$, written as a reduced fraction. (1)

2. The arrow maze at right is traversed as follows. Begin in square 7, and at each step move in the direction given by the arrow in your square to an adjacent square, then rotate the arrow in the square you just left 90° clockwise. Continue moving until you exit the maze by landing outside the grid. Which square is the last one you visit before exiting? (1)



3. For most values of b the equation $3(7+x) = b(7-x)$ has a solution. For example, when $b = 11$ we can solve to find $x = 4$. Find the only value of b for which this equation has no solution. (2)

4. Let us say that a finite sequence $a_1, a_2, \ldots, a_n$ of n integers is *thrilling* if the sum of any two consecutive terms of the sequence is a power of three. (The powers of three are 1, 3, 9, 27, ...) What is the length of the shortest thrilling sequence whose first term is 1 and whose last term is 100? (2)

5. In triangle ABC point M is the midpoint of $\overline{AB}$, point N is on side $\overline{BC}$ (but is not the midpoint), and segments $\overline{AN}$ and $\overline{CM}$ intersect at P. If we have $PA = 7$, $PC = 5$, and $PN = 3$ then compute length PM. (2)

6. There are several real numbers x such that $\log_2(x+2)$, $\log_4(3x+4)$, and $\log_8(7x+8)$ are three real numbers in arithmetic progression in the order listed. Determine the only irrational value of x having this property. (3)

7. Find the least positive integer n for which $201^n - 152^n$ is divisible by 2009. (3)

SCORE: